Doorway States and Intermediate Structure Phenomena

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Dedicated to Professor Dr. W. GENTNER *on the occasion of his 60th birthday*

The phenomenon of the distribution of a doorway state over many complicated compound nuclear states, i. e. the formation and properties of a so-called micro-giant resonance, are investigated analytically and numerically by means of a simple model. The distribution of the poles in the complex plane and the distribution of the residues of the S-matrix are obtained numerically and used to calculate the average cross-section. It is shown that the LANE-THOMAS-WIGNER theory applies, except for a possible correction to the shape of the resonance occuring in the cross-section averaged over the energy. This shape need not be describable by a LORENTZian. Estimates of the "spreading width" W and the "total width" W_0 are given for particular cases. It is noteworthy that the relationship $W_0 = W + \Gamma$ need not be correct even as an order-of-magnitude estimate, where Γ is the decay width of the doorway state.

Doorway states resulting from the two-body character of the nuclear forces and functioning as intermediaries between the continuum and the complicated compound nuclear states should manifest themselves through intermediate structure phenomena in nuclear excitation functions [1–5]. Recently, indications have been mounting for the existence of such phenomena, at least in certain regions of the periodic table [6, 7]. The isobaric analogue states, too, are manifestations of this phenomenon [8, 9]. In the present paper, we analyze the formal properties of doorway states and, in particular, investigate the intermediate structure phenomena induced by such states. This intermediate structure depends upon the various coupling constants (matrix elements) which enter into the theory.

A doorway state is defined [1–5, 10] as a bound state Φ of a model-HAMILTONian embedded in the continuous spectrum with the property that the matrix element of the complete HAMILTON-operator between Φ and the continuum is not small. Clearly, this definition is dependent both on the model-HAMILTONian and on the channel (the continuum) under consideration. In order for intermediate structure phenomena to exist, it is necessary that the density of doorway states be sufficiently small. Otherwise, the doorway states overlap and intermediate structure connected with individual doorway states gets lost. This condition (that the density of doorway states be small) is met in the light nuclei where we deal with isolated resonances for sufficiently low energies. In the medium-weight and heavy nuclei, intermediate structure phenomena associated with individual doorway states are expected to show up most strongly in reactions involving simple projectiles (nucleons) and target nuclei in the vicinity of closed shells. Also, such phenomena may appear favourably in particular ranges of excitation energies of the compound nucleus. However, it is possible that collective excitations may also function as doorway states [11, 12]. At any rate, it is clear that even isolated doorway states in medium-weight and heavy nuclei will in most cases not appear as individual resonances, but instead as "microgiant" resonances [13, 14] and it is this case that we are presently interested in. An

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isolated doorway state is "smeared out" over a large number of compound nuclear states. Thereby, the partial widths of these states relating to particular channels acquire coherence properties that lead to a microgiant resonance. The existence of such coherence properties is, in fact, a model-independent definition of a doorway state. In the following, we investigate this coherence phenomenon analytically and numerically. We are particularly interested in the increase of the "smearing" with increasing coupling between doorway state and the complicated compound states, and in the correlations in magnitude and sign of the residues of the poles of the S-matrix. The precise knowledge of these correlations is necessary for the calculation of the average cross-section, i. e. for the understanding of the intermediate structure phenomenon.

In sect. I, we define our assumptions and notation. We describe general properties of the doorway state which can be obtained analytically. In sect. II, we describe the calculation of average cross sections and the dependence of the resulting parameters on the input parameters of the theory. In sect. III, we give some of the results of extensive numerical calculations.

I. A Model for a Doorway State

We confine ourselves to one channel (elastic scattering only). The formalism employed here is similar to that of refs. [10] and [15] and will not be explained unless it differs from these references. Given a model HAMILTONIAN H_0 with one doorway state Φ and a large number of complicated states $\Phi_i^{(1)}$, $i=1, \ldots, N$ embedded in a continuum with scattering functions Ψ_E, we assume that the complete HAMILTONIAN H has the following real matrix elements

$$\begin{aligned} \langle \Psi_E | H | \Psi_{E'} \rangle &= E\,\delta(E-E'), \\ \langle \Psi_E | H | \Phi \rangle &= V_E, \\ \langle \Psi_E | H | \Phi_i^{(1)} \rangle &= 0, \\ \langle \Phi | H | \Phi \rangle &= \varepsilon, \\ \langle \Phi | H | \Phi_i^{(1)} \rangle &= V_i, \\ \langle \Phi_i^{(1)} | H | \Phi_k^{(1)} \rangle &= \varepsilon_i\,\delta_{ik}. \end{aligned} \tag{1}$$

We thus assume that H has been diagonalized in the subspace of functions $\Phi_i^{(1)}$ and Ψ_E. We also assume that the elastic scattering phase shift δ obtained from the asymptotic behaviour of the functions Ψ_E and defined in equ. (3.3) of ref. [10] is constant over the energy range of interest, and that consequently V_E can be approximated by a constant, V say.

[15] H.-A. WEIDENMÜLLER and K. DIETRICH, Nucl. Phys. (in the press).

The S-matrix (now called S-function as we deal with a one-channel problem) has $N+1$ poles, corresponding to the coupling of the N states $\Phi_i^{(1)}$ and the state Φ to the continuum. As shown in refs. [10] and [15], it has the general form

$$S = \exp(2\,i\,\delta) \cdot \left(1 - i \sum_{\alpha=1}^{N+1} \frac{\Gamma_\alpha}{E-\mu_\alpha}\right) \tag{2}$$

with

$$\sum_{\alpha=1}^{N+1} \Gamma_\alpha = 2\,\pi\,V^2; \quad \sum_{\alpha=1}^{N+1} \mu_\alpha = \sum_{i=1}^{N} \varepsilon_i + \varepsilon - i\,\pi\,V^2. \tag{3}$$

The complex energies μ_α are defined as the roots of the equation

$$D(E) = \begin{vmatrix} E-\varepsilon_1 & & & & -V_1 \\ & E-\varepsilon_2 & & 0 & -V_2 \\ & & \ddots & & \vdots \\ & 0 & & E-\varepsilon_N & -V_N \\ -V_1 & -V_2 & \cdots & -V_N & E-\varepsilon+i\,\pi\,V^2 \end{vmatrix} = 0\,. \tag{4}$$

This equ. can be transformed into

$$\prod_{i=1}^{N+1} (E-\varepsilon_i) = \sum_{i=1}^{N} V_i^2 \prod_{\substack{l=1 \\ l\neq i}} (E-\varepsilon_l) \tag{5}$$

with $\varepsilon_{N+1} = \varepsilon - i\,\pi\,V^2$. From equ. (2.6) of ref [15], it follows that

$$\Gamma_\alpha = 2\,\pi\,V^2 \frac{\prod_{i=1}^{N} (\mu_\alpha - \varepsilon_i)}{\prod_{\substack{\beta=1 \\ \beta\neq\alpha}}^{N+1} (\mu_\alpha - \mu_\beta)}\,. \tag{6}$$

By taking the derivative of $D(E)$ with respect to E at $E=\mu_\alpha$ and using $D(E) = \prod_{\alpha=1}^{N+1} (E-\mu_\alpha)$ and equ. (5), it can be shown that

$$\Gamma_\alpha = \frac{2\,\pi\,V^2}{1+\sum_{i=1}^{N} \frac{V_i^2}{(\mu_\alpha-\varepsilon_i)^2}}\,. \tag{7}$$

Then, Γ_α can simply be calculated once the roots μ_α of equ. (5) are known. Similarly, it is possible to show that

$$\operatorname{Im} \mu_\alpha = \frac{-\pi\,V^2}{1+\sum_{i=1}^{N} \frac{V_i^2}{|\mu_\alpha-\varepsilon_i|^2}}\,, \tag{8}$$

so that $\lim_{\mathrm{Im}\,\mu_\alpha \to 0} (\Gamma_a / -2\,\mathrm{Im}\,\mu_a) = 1$ as should be the case.

The equs. given above can be interpreted physically as follows. Let us first discuss the case where $V_i = 0$ for all i. Then, $\mu_i = \varepsilon_i$ for $i = 1, \ldots, N+1$, and $\Gamma_i = 0$ for $i \leqq N$. Furthermore, $\Gamma_{N+1} = 2\pi V^2$. The S-function has a single pole at a distance $\frac{1}{2}\Gamma_{N+1}$ from the real axis, corresponding to the doorway state. This pole manifests itself in the elastic scattering cross-section $\sigma(E)$ through the occurence of an isolated resonance. Here, $\sigma(E)$ is (except for factors not interesting here) given by

$$\sigma(E) = |S(E) - 1|^2 = 2(1 - \mathrm{Re}\,S(E)). \quad (9)$$

As we turn on the couplings V_i, the function $S(E)$ acquires $N+1$ poles, which according to equ. (8) are all situated below the real axis. The doorway-state pole moves toward the real axis. This follows from the second of equs. (3). At the same time, the strength $2\pi V^2$ with which the doorway state Φ is coupled to the continuum is spread over the $N+1$ resonances which now arise, according to the first of equs. (9). As long as the matrix elements V_i are sufficiently small, we may use perturbation theory to calculate μ_a and Γ_a and obtain

$$\mu_k = \varepsilon_k + \frac{V_k{}^2}{\varepsilon_k - \varepsilon_{N+1}}; \quad k = 1, \ldots, N,$$
$$\mu_{N+1} = \varepsilon_{N+1} + \sum_{k=1}^{N} \frac{V_k{}^2}{\varepsilon_{N+1} - \varepsilon_k}, \quad (10)$$

and

$$\Gamma_k = 2\pi V^2 \frac{V_k{}^2}{(\varepsilon_k - \varepsilon_{N+1})^2}, \quad k = 1, \ldots, N;$$
$$\Gamma_{N+1} = 2\pi V^2 - \sum_{k=1}^{N} \Gamma_k. \quad (11)$$

In this case, the spreading of the doorway state over the compound states is very small. We call this situation case a. It is remarkable to see that the perturbation-theoretical denominators occurring in equs. (10) and (11) contain both the positions and the widths of the unperturbed states. This shows that the quantity of interest for a perturbation-theoretical approach is not the difference of (real) energies between the states (this might become arbitrarily small for sufficiently high densities of the states $\Phi_i^{(1)}$), but the difference between the poles in the complex plane. The pattern resulting in the scattering amplitude for case a is that of a single pole providing the background (a broad resonance), superimposed upon which the N poles of the states $\Phi_i^{(1)}$ produce quick variations. In this case, we do not encounter a doorway state in the true meaning of the word: The state Φ decays back into the continuum long before it starts populating the states $\Phi_i^{(1)}$, so that, although it fulfills the formal definition of a doorway state, it actually blocks the entrance to the compound nucleus.

As we further increase the strength of the matrix elements V_i, the doorway state pole approaches the N other poles corresponding to the states $\Phi_i^{(1)}$, until it merges with them. Thereby, the doorway state Φ looses its identity and becomes spread over the $N+1$ compound nuclear resonances. We now consider a situation extremely opposite to case a, where the matrix elements V_i are large, and where the merger and the distribution of Φ over the $N+1$ resonances are so complete that each of the $N+1$ quantities μ_a has a very small imaginary part that can be treated in perturbation theory. In that case, the matrix occurring in equ. (4) without the term $i\pi V^2$ in the $(N+1, N+1)$ element can be diagonalized by a real orthogonal matrix O_{ik} with resulting real eigenvalues λ_i. The elements O^*_{iN+1} give the distribution of the doorway state over the $N+1$ eigenstates of the system; they will be assumed to have a Lorentzian distribution,

$$O^2_{iN+1} = \frac{W\,d}{2\pi[(\lambda_i - \varepsilon)^2 + \frac{1}{4}W^2]}, \quad i = 1, \ldots, N+1, \quad (12)$$

where W is the "spreading width" of the doorway state over the eigenstates, and d is the average distance between neighbouring eigenvalues λ_i. The quantity W has been estimated in refs. [4] and [16], these authors find

$$W \cong \frac{2\pi}{d} \overline{V^2} \quad (13)$$

where $\overline{V^2}$ is the average of $V_i{}^2$. Transforming the matrix in equ. (4) with O_{ik}, we obtain the new determinant

$$D(E) = |(E - \lambda_j)\,\delta_{jk} + \frac{i}{2} O_{j\,N+1} O_{k\,N+1} \Gamma| = 0. \quad (14)$$

If $W \gg \Gamma = 2\pi V^2$, perturbation theory in equ. (14) becomes, according to equ. (12), applicable and we obtain for the eigenvalues μ_a the expression

$$\mu_j = \lambda_j - \frac{i}{2} O^2_{jN+1} \cdot \Gamma; \quad j = 1, \ldots, N+1. \quad (15)$$

Correspondingly, we have in lowest order in Γ / W

$$\Gamma_j = O^2_{jN+1} \cdot \Gamma; \quad j = 1, \ldots, N+1. \quad (16)$$

[16] M. Danos and W. Greiner, Phys. Rev. **138**, B 876 [1965].

The case $W \gg \Gamma$, henceforth called case b, does lead to a true doorway-state phenomenon: While the population of the $N+1$ compound states is possible only through the doorway state Φ, each of these $N+1$ states is populated, and the original doorway state has totally disappeared. It is smeared out over the compound states and manifests itself only in a coherence property of the widths Γ_a, as shown by equs. (16) and (12). Unfortunately, case b seems as unrealistic as case a. Any intermediate case can only be treated numerically which is done in sect. III.

II. Average Cross Sections

The average cross section $\langle\sigma(E')\rangle_E$ is defined by

$$\langle\sigma(E')\rangle_E = \int_{E-L/2}^{E+L/2} \mathrm{d}E'\, \sigma(E')\, f(E'). \tag{17}$$

According to ref. [17], we choose

$$f(E') = \frac{I}{\pi}\left[(E'-E)^2 + I^2\right]^{-1} \tag{18}$$

and take $L \to \infty$. Using the second of equs. (9), and equ. (2) we obtain [17]

$$\langle\sigma(E')\rangle_E = (\cos 2\delta)\cdot\left(\sum_{\alpha=1}^{N+1} \frac{i\,\Gamma_a}{E-\mu_a+i\,I} + \text{c. c.}\right) + (\sin 2\delta)\cdot\left(\sum_{\alpha=1}^{N+1}\frac{\Gamma_a}{E-\mu_a+i\,I} + \text{c. c.}\right) + 2(1-\cos 2\delta). \tag{19}$$

We calculate the expression (19) for cases a and b. We make the usual approximations, i. e. $d \ll I$ where for case b the quantity d is defined as the average distance of the $\mathrm{Re}\,\mu_a$. Then, the sums occurring in equ. (19) can be transformed into integrals. In case a we furthermore assume that $|\mathrm{Im}\,\mu_a| \ll I$ for $\alpha \neq N+1$, that $\Gamma \gg I$ and that $V_i^2 = \alpha \ll \Gamma$ with α independent of i. Under these approximations, we obtain

$$\sum_{\alpha=1}^{N+1} \frac{\Gamma_a}{E-\mu_a+i\,I} \longrightarrow \frac{\Gamma}{E-\varepsilon+\frac{1}{2}\,i\,\Gamma} \tag{20 a}$$

In other words, the average cross section obtained from equs. (19) and (20) displays the single doorway state resonance, while the superimposed oscillations have disappeared. For case a this should be expected. For case b, we use the assumption $W \gg I \gg |\mathrm{Im}\,\mu_a|$ and obtain

$$\sum_{\alpha=1}^{N+1} \frac{\Gamma_a}{E-\mu_a+i\,I} \longrightarrow \frac{\Gamma}{E-\varepsilon+\frac{1}{2}\,i\,W}. \tag{20 b}$$

Again, because of $\Gamma \ll W$ this ought to be expected, but yields a different expression for the average cross section than that obtained from equ. (20 a). The case intermediate between a and b cannot be treated analytically unless several assumptions are introduced. We postpone a justification of these assumptions to the next section. We do not now assume that $W \gg \Gamma$. However, we are interested in a case where the doorway state pole has already merged with the other N poles. In this case, the quantities Γ_a are no longer real. For simplicity, we assume that the quantities V_i^2 are distributed symmetrically, i. e. that $V_i^2 = V_{N+1-i}^2$. It then follows from equ. (5) that the μ_a's are distributed symmetrically about ε, and from equ. (7) that the quantities $\mathrm{Re}\,\Gamma_a$ are distributed symmetrically, the quantities $\mathrm{Im}\,\Gamma_a$ skew-symmetrically about ε. For $I \gg d$, we can again replace summations by integrals and therefore deal with a function $\Gamma(E)$ obtained from the quantities Γ_a. We describe $\Gamma(E)$ by the formula

$$\frac{1}{\mathrm{d}}\,\Gamma(E) = \Gamma\,\frac{W_0}{2\pi}\,\frac{1}{(E-\varepsilon)^2+\frac{1}{4}W_0^2} + i\,\Gamma_1\,\frac{W_1^2}{2\pi}\,\frac{\mathrm{d}}{\mathrm{d}E}\,\frac{1}{(E-\varepsilon)^2+\frac{1}{4}W_1^2}, \tag{21}$$

which is in accord with the first of equs. (3). Under the further assumption $|\mathrm{Im}\,\mu_a| \ll I$ we can calculate the average cross section, we obtain for $I \ll W_0, W_1$

$$\sum_{\alpha=1}^{N+1} \frac{\Gamma_a}{E-\mu_a+i\,I} \to \frac{\Gamma}{E-\varepsilon+\frac{1}{2}\,i\,W_0} - i\,\frac{\Gamma_1\,W_1}{(E-\varepsilon+\frac{1}{2}\,i\,W_1)^2}. \tag{22}$$

In order to discuss qualitatively the change resulting from equ. (22) in the average cross section, we first write the average cross section for case b, combining equs. (19) and (20 b)

$$\langle\sigma(E')\rangle_E = \frac{\Gamma\,W}{(E-\varepsilon)^2+\frac{1}{4}W^2}\cos 2\delta + \frac{2\,\Gamma(E-\varepsilon)}{(E-\varepsilon)^2+\frac{1}{4}W^2}\sin 2\delta + 2(1-\cos 2\delta). \tag{23}$$

We now assume $W_0 \approx W_1$ and define $x = \Gamma_1/\Gamma$. Then, we obtain from equ. (22) a formula similar to equ. (23), except for the following changes. The quantity W in equ. (23) has to be replaced by W_0.

[17] G. Brown, Rev. Mod. Phys. 31, 893 [1959].

The term in front of cos 2 δ has to be multiplied by

$$A(E) = (1+2\,x)\,\frac{(E-\varepsilon)^2+\frac{1}{4}W_0^2\left(\frac{1-2\,x}{1+2\,x}\right)}{(E-\varepsilon)^2+\frac{1}{4}W_0^2}\,, \tag{24 a}$$

the term in front of sin 2 δ by

$$B(E) = \frac{(E-\varepsilon)^2+\frac{1}{4}W_0^2(1-4\,x)}{(E-\varepsilon)^2+\frac{1}{4}W_0^2}\,. \tag{24 b}$$

It can easily be seen that both $A(E)$ and $B(E)$ lead to an effective widening (narrowing) of the resonance for $x>0$ $(x<0)$. This statement is of interest because from the measured shape of an intermediate structure resonance, one would like to extract information regarding the quantities Γ and W, the latter being defined in equ. (13). The questions to be answered by the numerical calculations can thus be summarized as follows. **i)** For which values of W [equ. (13)] does the merger between the doorway state and the complicated states take place? **ii)** Is the description (21) for the distribution of the Γ_α's realistic? **iii)** What are the values for the quantities W_0, W_1, Γ_1 in dependence on Γ and W? Can we put $W_0 \cong W_1$, and is it reasonable to write $W_0 \cong \Gamma + W$?

III. Numerical results

We have investigated numerically the solutions of the equs. (5) and (7), using the following input data. We put $\varepsilon_{N+1} = 25.5 - i\,\beta$ and $N = 50$, so that $\varepsilon_j = j$, $j = 1, \dots, 50$. All matrix elements V_i^2 were assumed to be equal, $V_i^2 = \alpha$. The solutions thus depend upon the two parameters α and β. In order to have $N \gg \beta \gg d \approx 1$, we have chosen $\beta = 8$, which corresponds to a width $\Gamma = 16$. We have also investigated the general behaviour of the solutions for different values of β, but those obtained for $\beta = 8$ are typical so that we restrict our discussion to them. The quantity W defined by equ. (13) is given by $\sim 2\,\pi\,\alpha$, since $d \cong 1$ with our choice of parameters. The merger between the doorway state and the many complicated states proceeds as follows: For $\alpha = 1$, $W \cong 6.28$ the doorway state is still distinct from the complicated states, it is situated at a distance of 5.3 from the real axis, whereas the neighbouring poles have distances of about 0.2 from the real axis. For $\alpha = 2$, $W \cong 12.57$ the situation has not changed qualitatively, the corresponding figures are now 2.0 and 0.3, respectively.

Re μ_α	Im μ_α	Re Γ_α/Γ	Im Γ_α/Γ
50.180	−0.1115	+0.00713	−0.01246
49.165	820	709	796
48.161	763	684	715
47.160	−0.0753	+0.00681	−0.00699
46.161	762	690	740
45.163	785	707	738
44.167	816	730	777
43.170	855	758	828
42.174	901	791	889
41.179	955	830	961
40.185	1017	874	1046
39.190	1088	924	1146
38.197	1168	983	1261
37.204	1258	1052	1396
36.211	1361	1135	1555
35.220	1478	1236	1741
34.229	1611	1363	1961
33.240	1764	1529	2224
32.252	1939	1754	2539
31.266	2141	2074	2916
30.283	2376	2550	3359
29.305	2647	3294	3843
28.334	2953	4475	4249
27.375	3279	6280	4192
26.430	3557	8421	2895
25.500	3675	9527	0
24.569	−0.3557	+0.08425	+0.02885
23.626	3278	6274	4197
22.666	2953	4479	4247
21.695	2647	3291	3845
20.717	2376	2552	3358
19.734	2142	2073	2917
18.748	1939	1754	2539
17.760	1764	1528	2224
16.770	1611	1363	1961
15.781	1478	1236	1741
14.789	1361	1135	1555
13.796	1258	1052	1396
12.803	1168	983	1261
11.810	1088	924	1146
10.815	1017	874	1046
9.821	955	820	961
8.826	901	791	889
7.830	855	758	827
6.833	816	730	777
5.837	785	707	738
4.839	762	690	711
3.840	753	681	699
2.839	763	684	715
1.835	820	709	796
0.820	1115	713	1246

Table 1. Distribution of the poles μ_α and residues Γ_α/Γ of the S-function.

For $\alpha = 3$, we have $W \cong 18.85$, so that $W \cong \Gamma = 16.0$, and the merger is complete. Table 1 shows in the first and second column the distribution of the real and imaginary parts of the quantities μ_β, $\beta = 1, \dots, 51$ [see equs. (2) to (4)] for $\alpha = 3$, and in the third and fourth column the values of the real and imaginary parts of the quantities Γ_β/Γ [see equ. (7)] for $\beta=1,\ldots,51$ and $\alpha=3$. This result (as well as the other cases investigated numerically) shows that the merger between the doorway state pole and the other poles becomes complete at values of $W \cong \Gamma$.

We now turn to the distribution of the quantities Γ_β/Γ (Table 1, third and fourth column). In the first place, it is interesting to notice that Γ_β/Γ is not related to Im μ_β in any simple way. This should be expected since we deal with *overlapping* resonances. A comparison between column three of that table and equ. (21) shows that is not possible to fit the distribution exactly by a LORENTZian. This is because according to equ. (21) the real part of Γ_β/Γ should be given by

$$d\frac{W_0}{2\pi}\cdot\frac{1}{(E_\beta-\varepsilon)^2+\frac{1}{4}W_0^2}.$$

This expression contains only one parameter, W_0, which is determined by the height of the distribution at $E_\beta=\varepsilon=25.5$. We obtain $W_0=6.7$, and with this value $\Gamma_\beta/\Gamma=0.031$ at $E=31.266$, whereas the value from Table 1 is 0.021. On the other hand, at $E=50.180$ we obtain $\Gamma_\beta/\Gamma=0.0017$, in contrast to the value 0.0071 from Table 1. Probably part of this discrepancy is due to the fact that we have used a finite range of energies, whereas equ. (21) applies to an infinitely large energy interval. It is clear, however, that a LORENTZian distribution for Re Γ_β applies only rather approximately, in particular since it also depends upon the distribution of the quantities V_i^2. A better fit to the wing of the distribution is obtained by decreasing W_0; then, however, the values obtained near the center of the distribution are too small. The quantity W_0 may be called the spreading width, it is extremely interesting that the relationship $W_0=\Gamma+W$ applies not even as an order-of-magnitude estimate, since $W_0=3\ldots7$ is the total width entering into the formula for the averaged cross-section, equ. (23), and $\Gamma \cong W \cong 16\ldots18$.

The distribution of the imaginary part of Γ_β/Γ (column four of Table 1) can be fitted more easily with the formula (21), simply because we now have two parameters at our disposal, the width W_1 and the height Γ_1 of the distribution. A rough fit to the distribution (done by hand) yields $W_1=3.3$, which in view of the bad fit obtained for Re Γ_β/Γ agrees rather well with $W_0=6.7$, and $\Gamma_1/\Gamma = -0.045$. This shows that the quantity $x=\Gamma_1/\Gamma$ appearing in equ. (24) is exceedingly small, in spite of the fact that Im Γ_β is of the same order of magnitude as Re Γ_β.

In addition, we wish to point out that the distribution of the quantities μ_α was not in all cases as smooth as that shown in Table 1. If one increases the coupling α further and considers, f. i., a value $\alpha=8.0$, one finds that the poles μ_β tend to repel each other and lie no longer on a smooth curve. Whether or not this effect was caused by the finite energy interval chosen could not be decided within the frame of the present investigation.

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